

# Electroweak Theory without a Higgs potential: Radiative Effects

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## Abstract

We examine the one loop effective potential in a recently proposed (by Chernodub et. al.) alternative approach to mass generation in the Higgs-gauge sector of the electroweak theory, which does not make use of a classical Higgs potential. We show that the interpretation given by these authors, of the Higgs boson as the conformal degree of freedom in a background conformal gravity theory, is invalidated because genuine one loop radiative effects cancel the local functional measure of the Higgs field taken to be the basis of this interpretation. Functional evaluation of the one loop effective Higgs potential leads to a minimum away from the origin, thereby providing yet another radiative mass generation scheme for weak gauge bosons. In arriving at the one loop effective potential we make use of the gauge free formulation of electrodynamics introduced by us earlier, which obviates any need for gauge fixing.

## 1 Introduction

Recently a new approach to the Higgs phenomenon of mass generation in the gauge boson-Higgs boson sector of standard electroweak gauge theory has been proposed [1], [2], [3], [4] together with a novel interpretation of the Higgs scalar degree of freedom. Beginning with a standard  $SU(2) \times U(1)$  gauge theory augmented by a Higgs action, but *without* a Higgs potential, field redefinitions are performed whereby all fields are rendered completely *inert* under  $SU(2)$  gauge transformations.<sup>1</sup> As a result of field redefinitions, the theory has a remaining Abelian gauge invariance under  $U(1)_{em}$  with the photon being left over as the only massless gauge field. The gauge neutral modulus of the Higgs field couples to the  $W$  and  $Z$  bosons in the standard manner and can indeed provide masses to them if it picks up a vacuum expectation value. Unlike in the standard Higgs mechanism which employs a (unstable) Higgs potential for this purpose, the Higgs degree of freedom here is given a novel interpretation: it is the conformal degree of freedom in a conformally flat background gravity theory, such that its vacuum value is fixed by its large distance (cosmological) behaviour. This interpretation derives from the altered functional measure for the Higgs modulus field because of field redefinitions: it is *local* in nature. Indeed, this is a fascinating alternative to the standard electroweak Higgs mechanism if it survives quantization. The possibility that this may unfortunately *not* be the case has already been considered in [5]. In this paper, we explicitly consider perturbative radiative effects in the entire scenario to study this question in more detail. We consider the one loop effective potential of the theory and find that the local terms in the functional measure in fact do *not* survive quantization : they are exactly cancelled by genuine radiative terms arising from the functional evaluation of the one loop effective potential. The novel interpretation is thus subject to modification in a full quantum field theory treatment.

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<sup>1</sup>This is perhaps related to the gauge-free electrodynamics [6] proposed by us elsewhere, generalized to the context of a non-Abelian gauge theory with Higgs couplings.

Rather than attempting to resurrect the interpretation, we probe the question as to whether an effective Higgs potential might emerge *radiatively*. The one loop effective potential we compute indeed has a minimum away from the origin, determined by the renormalization scale and the gauge couplings. There is thus adequate structure at the quantum level of the theory to generate gauge boson (and Higgs) masses in terms of its parameters. We compare the results of the Higgs-vector mass ratio obtained here to that obtained in the standard Coleman-Weinberg approach [7] where conventional Higgs mechanism is implemented radiatively.

The plan of the paper is as follows: in section 2 the field redefinitions employed in [2] that render all fields free of  $SU(2)$  gauge transformations are discussed briefly. These are important because they enable us to work with  $SU(2)$  invariant fields, thereby obviating the need to perform  $SU(2)$  gauge fixing. We extend these field redefinitions to the  $U(1)_{em}$  invariant residual theory as well, following our earlier work [6], so that all fields are rendered manifestly  $U(1)$  invariant, thereby avoiding any gauge fixing at all. This is followed in section 3 with a computation of the one loop effective Higgs potential, using the same local measure discerned in [2]. It is shown how the effects of the local measure are cancelled by radiative terms in the effective potential. Appropriate renormalizations are performed to yield a renormalized one loop effective Higgs potential. In section 4, it is shown that the effective Higgs potential possesses an absolute minimum away from the origin in Higgs field space, and discuss the spectrum of particles around this minimum. The concluding section (5) contains a discussion of our results.

## 2 New variable form of $SU(2) \times U(1)$ theory

This section is entirely based on [2]. The gauge-Higgs sector of the standard electroweak theory contains the  $SU(2)$  gauge field  $\mathbf{B}_\mu = B_\mu^a t^a$ ,  $a = 1, 2, 3$ , the  $U(1)$  gauge field  $Y_\mu$  and the Higgs  $SU(2)$  doublet  $\Phi$ , transforming under  $SU(2) \times U(1)$  gauge transformations as

$$\begin{aligned} \mathbf{B}_\mu &\rightarrow \mathbf{B}_\mu^{(\Omega)} = \Omega \mathbf{B}_\mu \Omega^{-1} - \partial_\mu \Omega \Omega^{-1} \\ Y_\mu &\rightarrow Y_\mu^{(\omega)} = Y_\mu + \partial_\mu \omega \\ \Phi &\rightarrow \Phi^{(\Omega)} = \Phi \Omega, \quad \Phi \rightarrow \Phi^{(\omega)} = \Phi \exp i\omega. \end{aligned} \quad (1)$$

The Lagrange density for this sector of the electroweak theory is

$$\mathcal{L} = (\nabla_\mu \Phi \nabla^\mu \Phi) - \frac{1}{4g^2} \text{tr} \mathbf{B}_{\mu\nu}^2 - \frac{1}{4g'^2} Y_{\mu\nu}^2, \quad (2)$$

where

$$\begin{aligned} \nabla_\mu \Phi &= \partial_\mu \Phi + \frac{i}{2} Y_\mu \Phi + B_\mu^a t^a \Phi \\ B_{\mu\nu}^a &= \partial_\mu B_\nu^a - \partial_\nu B_\mu^a + \epsilon_{abc} B_\mu^b B_\nu^c \\ Y_{\mu\nu} &= \partial_\mu Y_\nu - \partial_\nu Y_\mu \end{aligned}$$

with  $t^a = \frac{i}{2} \tau^a$ ,  $\tau^a$  Pauli matrices,  $g$  and  $g'$  are the coupling constants.

The essential feature of this approach is the ‘polar decomposition’ of the complex scalar doublet into two parts.

$$\Phi = \frac{1}{\sqrt{2}} \rho \chi, \quad (3)$$

where,  $\rho$  is a real positive scalar (modulus) field is completely gauge-inert, while the ‘phase’ part  $\chi$  carries all the gauge transformation properties of  $\Phi$ . Now, one introduces the matrix

$$g = \begin{pmatrix} \chi_1 & -\bar{\chi}_2 \\ \chi_2 & \bar{\chi}_1 \end{pmatrix} \quad (4)$$

with a normalisation  $(\chi, \chi) = \bar{\chi}_1 \chi_1 + \bar{\chi}_2 \chi_2 = 1$  (which defines the group manifold of  $SU(2)$ ), it is easy to verify [2] that  $g$  is unimodular and unitary. It stands to reason that  $g \in SU(2)$  so that under an  $SU(2)$  gauge transformation

$$g \rightarrow g^{(\Omega)} = \Omega g. \quad (5)$$

However, since  $\chi_i$  and  $\bar{\chi}_i$  have different weak hypercharges, under a  $U(1)$  gauge transformation,  $g \rightarrow g^{(\omega)} = g e^{i\omega\tau_3}$

The covariant derivative of  $g$  is given by

$$\nabla_\mu g = \partial_\mu g + \frac{i}{2} Y_\mu g \tau_3 + \mathbf{B}_\mu g. \quad (6)$$

Defining the new Yang Mills triplet  $\mathbf{W}_\mu = W_\mu^a t^a$  as

$$\mathbf{W}_\mu \equiv g^\dagger (\mathbf{B}_\mu + \partial_\mu) g \quad (7)$$

it is easy to see that under an  $SU(2) \times U(1)$  gauge transformations,

$$\begin{aligned} \mathbf{W}_\mu^{(\Omega)} &= \mathbf{W}_\mu \\ \mathbf{W}_\mu^{(\omega)} &= e^{-i\omega\tau_3} \mathbf{W}_\mu e^{i\omega\tau_3} + i\tau_3 \partial_\mu \omega. \end{aligned} \quad (8)$$

These fields  $\mathbf{W}_\mu$  are thus explicitly  $SU(2)$  gauge invariant, even though they have nontrivial  $U(1)$  gauge transformations. One defines the linear combinations

$$\begin{aligned} Z_\mu &\equiv Y_\mu + W_\mu^3 \\ A_\mu &\equiv \frac{1}{g^2 + g'^2} (g'^2 W_\mu^3 - g^2 Y_\mu), \end{aligned} \quad (9)$$

where, the vector field  $Z_\mu$  is manifestly  $SU(2) \times U(1)$  invariant, and the  $A_\mu$  field transforms under  $U(1)$  as  $A^{(\omega)} = A_\mu - 2\partial_\mu \omega$ . The charged combinations  $W_\mu^\pm \equiv W_\mu^1 \tau_1 \pm W_\mu^2 \tau_2$  are  $SU(2)$  gauge invariant, but carry indicated charges under  $U(1)_{em}$  gauge transformations under which  $A_\mu$  transforms as the photon field, with the electronic charge being  $e^{-2} \equiv g^{-2} + g'^{-2}$ .

The entire gamut of field redefinitions leave only a  $U(1)_{em}$  gauge theory with the photon field being the sole gauge connection; the Yang Mills connections have been rendered entirely gauge free under  $SU(2)$  gauge transformations, and behave as charged (or neutral) vectorial matter fields under electromagnetism. The theory is described by the Lagrange density

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \partial_\mu \rho \partial^\mu \rho + \frac{\rho^2}{8} (Z_\mu^2 + W_\mu^+ W^{\mu,-}) - \frac{1}{4g^2} (\nabla_\mu W_\nu^+ - \nabla_\nu W_\mu^+) (\nabla^\mu W^{\nu,-} - \nabla^\nu W^{\mu,-}) \\ & - \frac{1}{4(g^2 + g'^2)} Z_{\mu\nu}^2 - \frac{1}{4e^2} A_{\mu\nu}^2 - \frac{2}{4g^2} H_{\mu\nu} (A^{\mu\nu} + e^2 Z^{\mu\nu}) - \frac{1}{4g^2} H_{\mu\nu}^2, \end{aligned} \quad (10)$$

where

$$\begin{aligned} Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu \\ A_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \\ W_{\mu\nu}^3 &= \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3 \\ H_{\mu\nu} &= \frac{1}{2i} (W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+) \\ \nabla_\mu W_\nu^\pm &= \partial_\mu W_\nu^\pm \pm i W_\mu^3 W_\nu^\pm. \end{aligned} \quad (11)$$

The question is : does this theory generate a mass for the  $W_\mu^\pm$ , the  $Z_\mu$  and the  $\rho$  fields, as is achieved in the standard Higgs mechanism by means of a Higgs potential with degenerate minima ? In other words,

what is the scale of the vacuum expectation value  $\rho$  here, since there is no Higgs potential to generate that scale? In [1] the Higgs modulus field  $\rho$  is interpreted, because of the appearance of the local  $\rho^2 \mathcal{D}\rho$  factor that appears in the partition functional integral, as the conformal factor of a background conformally flat spacetime. The vacuum value of  $\rho$  is related to its asymptotic value in this spacetime, and is supposed to be determined cosmologically. Excitations around this vacuum value are of course to be interpreted as a new massless scalar field. So, a new perturbative mechanism to produce vector boson masses becomes available, as an alternative to the standard Higgs mechanism. The issue is: does this mechanism survive quantization?

Even though the theory has a residual  $U(1)_{em}$  gauge invariance, one can rewrite it explicitly in terms of entirely *gauge free* variables [6], so that no gauge fixing is at all necessary to evaluate the partition function. We begin by a radial decomposition of the charged weak vector boson fields

$$W_\mu^\pm = w_\mu \exp \pm i\theta^{(\mu)}, \text{ no sum on } \mu \quad (12)$$

which implies that under  $U(1)$  gauge transformations

$$[w_\mu]^{(\omega)} = w_\mu, \quad [\theta^{(\mu)}]^{(\omega)} = \theta^{(\mu)} + 2\omega. \quad (13)$$

One can think of  $w^\mu$  as the component of the charged vector boson carrying only the *spin* while  $\theta^{(\mu)}$  is the *charge* mode, thus affecting a ‘separation of the charge and spin modes’. It follows that

$$\nabla_\mu W_\nu^\pm = \left[ \partial_\mu w_\nu \pm i w_\nu \left( \partial_\mu \theta^{(\nu)} - A_\mu - \frac{e^2}{g'^2} Z_\mu \right) \right] e^{i\theta^{(\nu)}} \quad (14)$$

The only quantity sensitive to  $U(1)_{em}$  gauge transformations is the phase factor; the gauge transformation parameter cancels between the  $\theta$  and  $A$  fields within the parantheses. However, all fields can now be expressed explicitly in terms of entirely gauge free fields except the phase factor which indeed must carry the full burden of gauge transformations, through the field redefinitions [6].

$$\begin{aligned} \Theta^{(\mu)} &\equiv \theta^{(\mu)} - 2a \\ \mathbf{A}_\mu &\equiv A_\mu - 2\partial_\mu a, \end{aligned} \quad (15)$$

where,  $a(x) \equiv \int d^4x' G(x - x') \partial' \cdot A(x')$  is a scalar field giving the longitudinal mode of  $A_\mu(x)$  with  $G(x - x')$  being the Green’s function for the d’Alembertian. As a result of these field redefinitions, the kinetic energy of the charged vector bosons (and indeed the entire Lagrange density) is rendered free of *all* local gauge transformations. The former assumes the form

$$\begin{aligned} \nabla_{[\mu} W_{\nu]}^+ \nabla^{[\mu} W^{\nu]-} &= \frac{1}{2} \sum_{\mu, \nu=0}^3 \{ [\partial_\mu w_\nu \partial^\mu w^\nu + w_\nu (\tilde{A}_\mu^{(\nu)} + \frac{e^2}{g'^2} Z_\mu) w^\nu (\tilde{A}^{\mu(\nu)} + \frac{e^2}{g'^2} Z^\mu)] \\ &- \cos \Theta^{(\mu\nu)} [\partial_\mu w_\nu \partial^\nu w^\mu + w_\nu (\tilde{A}_\mu^{(\nu)} + \frac{e^2}{g'^2} Z_\mu) w^\mu (\tilde{A}^{(\mu)\nu} + \frac{e^2}{g'^2} Z^\nu)] \\ &- \sin \Theta^{(\nu\mu)} [\partial_\nu w_\mu \partial^\mu w^\nu (\tilde{A}^{(\nu\mu)} + \frac{e^2}{g'^2} Z^\mu)] \} \end{aligned} \quad (16)$$

where,  $\tilde{A}_\nu^{(\mu)} \equiv \mathbf{A}_\nu - \partial_\nu \Theta^{(\mu)}$  and  $\Theta^{(\mu\nu)} \equiv \Theta^{(\mu)} - \Theta^{(\nu)}$

Since the  $W^\pm$  carry electric charge  $\pm 1$ , one can in fact choose the phases  $\Theta^{(\mu)}$  to be the same, independent of the  $\mu$ , without any loss of generality. With this choice, eq. (16) simplifies considerably

$$\begin{aligned} \nabla_{[\mu} W_{\nu]}^+ \nabla^{[\mu} W^{\nu]-} &= w_{\mu\nu}^2 + \frac{1}{2} w^2 \left( \mathbf{A} - \partial\Theta + \frac{e^2}{g'^2} Z \right)^2 \\ &- \frac{1}{2} \left[ w \cdot \left( \mathbf{A} - \partial\Theta + \frac{e^2}{g'^2} Z \right) \right]^2, \end{aligned} \quad (17)$$

where,  $w_{\mu\nu} \equiv 2\partial_{[\mu}w_{\nu]}$ . This equation exhibits the  $U(1)_{em}$  gauge freedom of the fields manifestly, and also shows explicitly the coupling of the charged vector boson modes to the physical  $U(1)$  photon vector potential.

### 3 One Loop Effective Potential

We now turn to the question of the perturbative quantum behaviour of the theory. To study this question, we consider the one loop effective potential of the theory given by (10), and investigate if it has a nontrivial minimum driven by infrared instabilities as in the Coleman-Weinberg mechanism [7]. We do not use the new interpretation given in [1]. The issue then amounts to investigating the possibility of radiative generation of a Higgs potential (not just a mass term), with a self-coupling *determined* in terms of the gauge couplings. Since we are interested in only one-loop calculations we drop all the cubic and quartic interaction terms from the Lagrangian in (10) since they do not contribute at the one loop level. This is easy to see by simply drawing all possible one loop graphs with  $\rho$  external lines: none of them have the vertices that are being discarded here. One is thus dealing with the truncated Lagrangian relevant for one-loop calculations,

$$\begin{aligned} \mathcal{L}_{trun} = & \frac{1}{2}\partial_\mu\rho\partial^\mu\rho + \frac{\rho^2}{8}(Z_\mu^2 + w_\mu^2) - \frac{1}{4g^2}w_{\mu\nu}^2 \\ & - \frac{1}{4e^2}\mathbf{A}_{\mu\nu}^2 - \frac{1}{4(g^2 + g'^2)}Z_{\mu\nu}^2, \end{aligned} \quad (18)$$

where, the physical photon field is divergenceless  $\partial \cdot \mathbf{A} = 0$  as discussed in [6].

The generating functional of all Feynman graphs is given by

$$\begin{aligned} Z[J, \mathbf{J}_A, \mathbf{J}_Z, \mathbf{J}^+, \mathbf{J}^-] = & \int d\mu \delta[\partial \cdot \mathbf{A}] \exp i \int d^4x \mathcal{L}_{trun} \\ & \cdot \exp i \int d^4x (J\rho + \mathbf{J}_A \cdot \mathbf{A} + \mathbf{J}_Z \cdot Z + \mathbf{J}_w \cdot w + J_\Theta \Theta) \end{aligned}$$

where the measure

$$d\mu = Det \rho^2 \mathcal{D}\rho^2 \mathcal{D}\mathbf{A} \mathcal{D}a \mathcal{D}Z \mathcal{D}w \mathcal{D}\Theta \mathcal{D}g$$

where  $\mathcal{D}g$  is the  $SU(2)$  group volume and  $\mathcal{D}a$  is that of the  $U(1)_{em}$ . Since the action is manifestly independent of the fields characterizing these group volumes, they can be factored out of the functional integral and discarded as irrelevant multiplicative factors. Note however the *local* measure associated with the Higgs field  $\rho$ ; this implies that  $\rho$  is not an usual scalar field, as pointed out in [1], [2]. It appears to behave like a *dilaton* field which might acquire a vacuum value from cosmological sources. However, this local measure undergoes a precise cancellation, as we shall show shortly.

We calculate the one loop effective potential functionally from the generating functional using saddle-point method [8]. Shifting the positive real scalar field

$$\rho \rightarrow \rho_0 + \rho$$

where  $\rho_0$  is a spacetime constant chosen to be the saddle point, the one-loop effective potential becomes

$$\begin{aligned}
V_{eff}(\rho_0) = & 3i \int \frac{d^4 k}{(2\pi)^4} \ln(\rho_0) - \frac{i\hbar}{2} \int \frac{d^4 k}{(2\pi)^4} \ln(-k^2) \\
& - \frac{i}{2g^2} \int \frac{d^4 k}{(2\pi)^4} \ln \text{Det} \left[ \eta^{\mu\nu} \left( -k^2 + g^2 \frac{\rho^2}{4} \right) + k^\mu k^\nu \right] \\
& - \frac{i}{2(g^2 + g'^2)} \int \frac{d^4 k}{(2\pi)^4} \ln \text{Det} \left[ \eta^{\mu\nu} \left( -k^2 + (g^2 + g'^2) \frac{\rho^2}{4} \right) + k^\mu k^\nu \right] \\
& - \frac{i}{2e^2} \int \frac{d^4 k}{(2\pi)^4} \ln \text{Det} [-\eta^{\mu\nu} k^2] , 
\end{aligned} \tag{19}$$

where ‘Det’ means a determinant in the functional space. Upon evaluating the eigenvalues of the respective integrals we obtain

$$\begin{aligned}
V_{eff}(\rho_0) = & 3i \int \frac{d^4 k}{(2\pi)^4} \ln(\rho_0) - \frac{i\hbar}{2} \int \frac{d^4 k}{(2\pi)^4} \ln(-k^2) \\
& - 3i \int \frac{d^4 k}{(2\pi)^4} \ln \left( k^2 - g^2 \frac{\rho^2}{4} \right) - i \int \frac{d^4 k}{(2\pi)^4} \ln(\rho_0) \\
& - \frac{3i}{2} \int \frac{d^4 k}{(2\pi)^4} \ln \left( k^2 - (g^2 + g'^2) \frac{\rho^2}{4} \right) - 2i \int \frac{d^4 k}{(2\pi)^4} \ln \rho_0 . 
\end{aligned} \tag{20}$$

One significant outcome of the above expression is that the Jacobian contribution from the functional measure discerned in [1] (the 1st term in (20)) is exactly cancelled by two terms coming from the neutral and charged vector boson operators (the fourth and seventh terms respectively in (20)). Another fact of this result is photon part of the Lagrangian doesn't contribute to the one-loop effective potential since it is not coupled to any other field of the theory. Thus on integration it will give an irrelevant infinite constant which we have subtracted off. After introducing renormalizing counterterms and Wick rotating the contour of integration, the effective potential becomes

$$\begin{aligned}
V_{eff}(\rho_0) = & \frac{B}{2} \rho_0^2 + \frac{C}{4!} \rho_0^4 \\
& + \frac{3}{2} \int \frac{d^4 k_E}{(2\pi)^4} \ln \left( k_E^2 + (g^2 + g'^2) \frac{\rho^2}{4} \right) + 3 \int \frac{d^4 k_E}{(2\pi)^4} \ln \left( k_E^2 + g^2 \frac{\rho^2}{4} \right) , 
\end{aligned}$$

where B, C are the usual mass and coupling-constant renormalization counter terms. The counterterms are determined using the renormalization scheme

$$\left. \frac{d^2 V}{d\rho_0^2} \right|_{\rho_0=M} = 0 \tag{21}$$

and

$$\left. \frac{d^4 V}{d\rho_0^4} \right|_{\rho_0=M} = 0 \tag{22}$$

where the parameter  $M$  serves as a scale of the theory. The renormalized effective one loop Higgs potential is now given by

$$\begin{aligned}
V_{eff}(\rho_0) = & \frac{27(g^2 + g'^2)^2 M^2 \rho_0^2}{512\pi^2} + \frac{27g^4 M^2 \rho_0^2}{256\pi^2} \\
& + \left( \frac{3(g^2 + g'^2)^2 \rho_0^4}{1024\pi^2} + \frac{3g^4 \rho_0^4}{512\pi^2} \right) \left( \ln \frac{\rho_0^2}{M^2} - \frac{25}{6} \right) , 
\end{aligned} \tag{23}$$

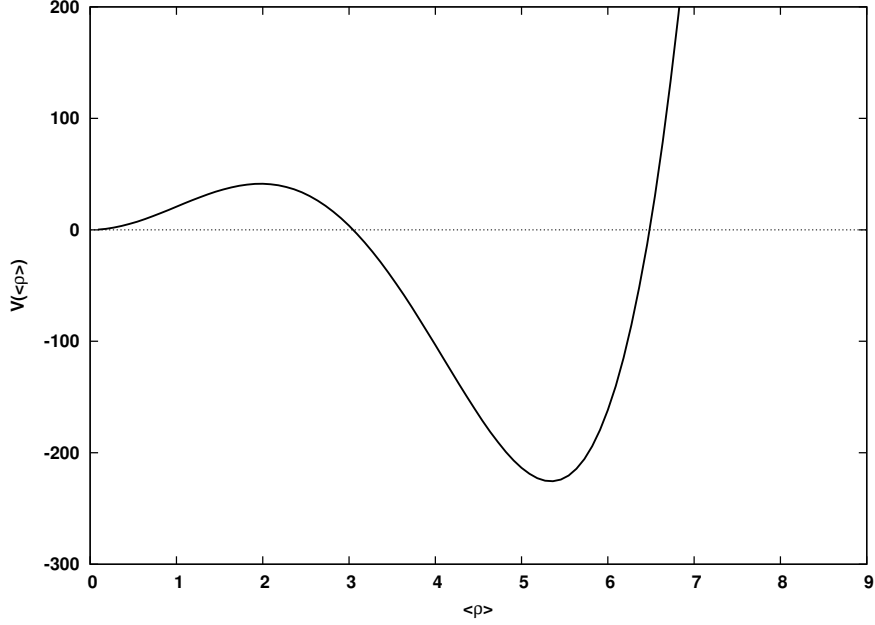


Figure 1: Plot of the Effective Potential as a function of  $\frac{\langle \rho \rangle}{M}$ .

Henceforth we drop the subscript 0 on  $\rho_0$ . The plot of effective potential (Fig. [1]) shows that it has three extrema in the physically interesting region i.e. in the region where  $\langle \rho \rangle > 0$ . Apart from a local minima at the origin the potential possesses a maxima at about  $\langle \rho \rangle \simeq 1.98M$ . The true minimum is around  $\langle \rho \rangle \simeq 5.34M$  for which  $\log(\langle \rho \rangle/M) \simeq 1.67$ .

The mass generated for the  $W^\pm$  bosons is  $(1/2)g\langle \rho \rangle$ , while for the  $Z$  bosons is  $(1/2)(g^2 + g'^2)^{1/2}\langle \rho \rangle$ . Thus, since  $\langle \rho \rangle = 246\text{Gev}$  reproduces the observed  $W$  and  $Z$  boson mass spectrum, this implies that one must make the choice  $M \sim 46\text{Gev}$ . This also corresponds to a  $\rho$  boson mass, computed as the curvature of the effective potential at the absolute minimum,

$$m_H^2 = 9 \frac{(g^2 + g'^2)^2 + 2g^4}{256\pi^2} \left( 3M^2 + \langle \rho \rangle^2 \ln \frac{(\langle \rho \rangle)^2}{M^2} - 3\langle \rho \rangle^2 \right) \quad (24)$$

The mass of the  $\rho$  field is computed to be 6.9 Gev : perhaps too light to be phenomenologically relevant as a standard Higgs field. It would be hard to identify  $\rho$  with a physical Higgs boson, the kind of which one expects to see at the Large Hadron Collider. This mechanism of mass generation must therefore be thought of as a toy model at this stage. However, it appears that LEP data has not completely ruled out a very light Higgs boson with a mass less than 10 Gev [9] which may mediate elastic scattering between light dark matter candidates.

One may compare the ratio of vector boson masses to the Higgs mass generated here with the corresponding result in the Coleman Weinberg framework [7]. The ratios are not very different, even though we did not have to resort to applying a ‘dimensional transmutation’ here in order to cast the scalar mass into a function entirely of the gauge couplings. This has been achieved here quite naturally, since there is no scalar self-coupling in the theory at the classical level.

## 4 Conclusion

While the imaginative interpretation offered in the incipient work of [1] appears to be untenable under radiative corrections, the one loop effective Higgs potential generated in the theory has the prospect of supplying the observed spectrum of weak vector boson masses, and possibly also a Higgs mass; however, the latter is too low so as to preclude the theory, if, as is commonly believed on the basis of LEP data, a Higgs boson is to be detected at the Large Hadron Collider with a mass just above the 140 GeV range. The theory in this paper has no self-coupling parameter, and the Higgs mass is completely determined by the gauge couplings with appropriate choice of the renormalization scale. The slightly disconcerting feature of this work is that the logarithm  $\log(\langle\rho\rangle)$  which features as an important dynamical factor in the loop expansion, has a value of  $O(1.67)$ . So long as one restricts oneself to low orders of perturbation theory, this does not pose a serious problem at the numerical level, even though theoretically it remains somewhat unsatisfactory.

An important issue is that of ‘naturalness’ of the scalar sector of the theory. Apart from the lacuna discussed above, this does *not* appear to be an issue, since there is no scalar self coupling. The renormalization scale  $M$  cannot arbitrarily slide to the GUT or the Planck scale without ruining the vector boson mass spectrum which is extremely well determined experimentally. The scalar mass is then *constrained* to be comparatively lower than the vector boson masses, and thus never requires fine tuning of dimensionless parameters.

The key question not addressed in this paper is of course the issue of fermion masses. One could add to the Lagrange density (2) fermionic gauge and Yukawa coupling terms where the Higgs vacuum expectation value produces fermion masses as in the standard electroweak theory. The radiatively generated Higgs vev then can be a source of fermion masses in the standard manner. However, in this case, the Yukawa couplings control fermion masses and mixings much like in the standard formulation, with no real economy in the size of the parameter space. The real challenge is to produce the fermion masses dynamically at the quantum level. Chiral symmetry prevents this from happening radiatively, so that the source of fermion masses might have to be nonperturbative if a scenario akin to the model under discussion ever becomes phenomenologically relevant.

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